

Identification of Observables for Quark and Gluon Orbital Angular Momentum

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A new debate has recently arisen on the subject of orbital angular momentum in QCD, in particular, on its observability, and on its partonic interpretation. Orbital momentum can be defined in QCD using two different decomposition schemes that yield a kinetic and a canonical definition, respectively. We argue that kinetic orbital angular momentum is intrinsically associated with twist three generalized parton distributions, and it is therefore more readily observable, while, due to parity constraints, canonical angular momentum, if defined as suggested in the literature in terms of generalized transverse momentum distributions, cannot be observed in scattering processes involving a single hadronic reaction plane.

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The question of the observability of Orbital Angular Momentum (OAM) in QCD was recently addressed in Ref.[1]. The main thrust of the paper was to identify an observable from DVCS experiments for OAM as given by the second moment of the twist-3 GPD, G_2 [2, 4]. Alongside with this identification, some difficulties were pointed out which are inherent with the alternative definition of OAM in terms of Generalized Transverse Momentum Distributions (GTMDs), concerning especially the way these objects would be measured in deeply virtual exclusive scattering processes from the proton. This triggered a series of observations in Ref.[5], related to the treatment of parity transformations in Ref.[1], some of which we believe are ill-founded. This situation, given the important issues at stake, *i.e.* the definition of OAM in QCD, and the possibility of measuring it, necessitates therefore an additional explanation.

The recent discussion follows in the steps of a debate which was initiated previously (see Refs.[6, 7] for reviews) on the gauge invariant decomposition of the total quark and gluon angular momenta, J^q , and J^g , into their respective spin and orbital components. One of the results out of this discussion was that it became clear that OAM could be defined through a twist three contribution from the relation [2, 3],

$$\int dx x G_2^q(x, 0, 0) = \frac{1}{2} \left[- \int dx x (H^q(x, 0, 0) + E^q(x, 0, 0)) + \int dx \tilde{H}^q(x, 0, 0) \right] \rightarrow \int dx x G_2^q(x, 0, 0) = -L^q, \quad (1)$$

where G_2^q is a specific twist three Generalized Parton Distribution (GPD) appearing in the parametrization of the quark-quark correlation function [2, 4, 8, 9] (G_2 was renamed \tilde{E}_{2T} in the full classification of GPDs given in Ref.[10]); H^q , E^q , and \tilde{H}^q are the twist two GPDs contributing to the observables for Deeply Virtual Compton Scattering (DVCS) processes introduced in [11] (see reviews in Refs.[12, 13]. L^q is referred to as *kinetic* [7] or *mechanical* [6] OAM, it appears in the relation [11],

$$\frac{1}{2} \Delta \Sigma + L^q + J^g = \frac{1}{2}, \quad (2)$$

and it is at variance with the *canonical* OAM, $L_{can}^{q,g}$ which is defined through the decomposition [14],¹

$$\frac{1}{2} = \frac{1}{2} \Delta \Sigma + L_{can}^q + \Delta G + L_{can}^g. \quad (3)$$

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¹ In our previous paper we used the notation $\mathcal{L}_q = L_{can}^q$.

$L_q(x)$ and $L_{can}^q(x)$ admit the same Wandzura Wilczek (WW) part, $L_q^{WW}(x)$, while they differ in their genuine twist three contribution [3, 15],

$$L_q(x) = L_q^{WW}(x) + \bar{L}_q(x) \quad (4a)$$

$$L_{can}^q(x) = L_q^{WW}(x) + \bar{L}_{can}^q(x). \quad (4b)$$

In the WW limit the two OAM distributions coincide, their differences depend on final state interactions contained in this case in the genuine twist three terms. In particular, $\int dx \bar{L}_q(x) = 0$, while $\int dx \bar{L}_{can}^q(x) \neq 0$, in general, so that it contributes to the angular momentum sum rule within this specific decomposition.

Notwithstanding the notion that the GPDs that enter Eq.(1) can be observed by measuring specific DVCS asymmetries and cross sections, to validate this relation it is however necessary to identify processes where OAM can be observed directly through the twist three GPD, G_2 . This was done in Ref.[1] where, making use of the expressions from an extensive analysis of DVCS at twist three level performed in [8, 9], we were able to single out the helicity amplitudes combinations which contribute to the twist three GPD G_2 , and to connect this structure function to an observable, namely the $\sin 2\phi$ modulation in the longitudinal Target Spin Asymmetry (TSA), $A_{LU}^{\sin 2\phi}$ [1]. This term has already been measured, and found to be quite substantial at HERMES [16] and CLAS [17]. It is also presently being analyzed at Jefferson Lab [18].

On the other side, canonical OAM is constructed by parametrizing the unintegrated correlation function [14] in the following way [15, 19–21],

$$L_{can}^q = \langle p, \Lambda | \int dx^- d\mathbf{x}_\perp i\psi^\dagger(\mathbf{x} \times \nabla)^3 \psi | p, \Lambda \rangle = - \int dx d^2 k_T \frac{k_T^2}{M^2} F_{14}(x, 0, k_T^2, 0, 0), \quad (5)$$

F_{14} is a specific GTMD – or an unintegrated over intrinsic- k_T GPDs – appearing in the decomposition of the vector component of the unintegrated quark-quark correlation function at twist two [10]

$$\begin{aligned} W_{\Lambda\Lambda'}^{\gamma+}(x, \mathbf{k}_T, \xi = 0, \mathbf{\Delta}_T; \eta) &= \frac{1}{2P^+} \bar{U}(p', \Lambda') \left[\gamma^+ F_{11} + \frac{i\sigma^{i+} \Delta_T^i}{2M} (2F_{13} - F_{11}) + \frac{i\sigma^{i+} \bar{k}_T^i}{2M} (2F_{12}) + \frac{i\sigma^{ij} \bar{k}_T^i \Delta_T^j}{M^2} F_{14} \right] U(p, \Lambda) \\ &= \delta_{\Lambda, \Lambda'} F_{11} + \delta_{\Lambda, -\Lambda'} \frac{-\Lambda \Delta_1 - i\Delta_2}{2M} (2F_{13} - F_{11}) + \delta_{\Lambda, -\Lambda'} \frac{-\Lambda \bar{k}_1 - i\bar{k}_2}{2M} (2F_{12}) + \delta_{\Lambda, \Lambda'} i\Lambda \frac{\bar{k}_1 \Delta_2 - \bar{k}_2 \Delta_1}{M^2} F_{14}. \end{aligned} \quad (6)$$

The connection of L_{can}^q , with kinetic OAM was discussed in several papers [6, 7]. Indeed Eq.(5) provides a plausible, intuitive identification which is inferred from the definition of canonical OAM originally suggested in [14]. Nevertheless, the fact that one can consider matching OAM onto experimental observables, only through a specific off-forward unintegrated parton distribution, or GTMD, entails various complications, from questions on both its factorizability and renormalizability in QCD, to the definition of a deeply virtual scattering process which could be sensitive to F_{14} . Such complications are not present for the GPD, G_2 , although there exists no obvious, straightforward partonic interpretation of this twist three quantity.

In this paper we address once more these issues, with the aim of amending some of the statements in Ref.[5] which misrepresent, and somewhat muddle the important steps towards the observability of OAM given in our original work [1]. Our goal is to provide additional support for pursuing experiments sensitive to both canonical and kinetic OAM. Two of the outstanding questions that we address: *i*) the nature of the parton distributions being identified with OAM, including the relative quarks and protons spin configurations involved; *ii*) the possibility to observe such configurations.

The message we would like to convey with this paper is that a more profound physical understanding of OAM may emerge only by defining a way to measure it. None of the existing literature indicates or states sufficiently clearly either whether this goal can be met, or which experimental setup it would require.

In Ref.[1] we demonstrated that there was a fundamental reason behind the claim that it was “not known how to extract Wigner distributions or GTMDs from experiments” [19], namely we explained how this inherent difficulty was a consequence of parity constraints on the helicity amplitudes which enter the general cross section formulation [22, 23].

Differently from the Transverse Momentum Distributions (TMDs) and the Compton Form Factors (CFFs) which can be extracted from semi-inclusive and deeply virtual exclusive lepton nucleon scattering, GTMDs cannot be obtained

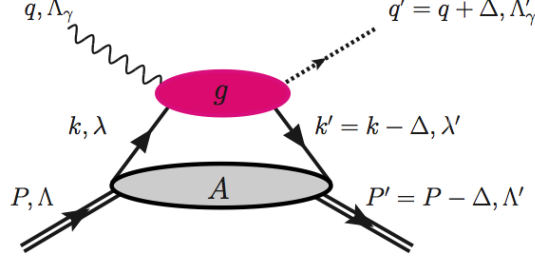


FIG. 1: The DVCS process (the crossed term diagram is not shown in the figure). The incoming (outgoing) particles momenta and helicities are labelled as follows: $k, \lambda(k', \lambda')$ for the quarks, $P, \Lambda(P', \Lambda')$ for the protons, and $q, \Lambda_\gamma(q', \Lambda'_\gamma)$ for the photons.

from two body scattering processes. In Figure 1 we illustrate for instance the DVCS process from which GPDs are extracted, namely

$$\gamma^* p \rightarrow \gamma p' \rightarrow (\gamma^* q \rightarrow \gamma q') \otimes (qp \rightarrow q' p')$$

which factors into γ^* -quark elastic scattering and two body quark-proton scattering (a similar illustration can be given for the TMD case). In such a process, it is always possible to define a Center-of-Mass (CoM) system where the two transverse momenta, \mathbf{k}_T and $\mathbf{\Delta}_T$ cannot be independent from one another (*i.e.* they belong to a single hadronic scattering plane). To extract F_{14} from experiment one first writes the helicity amplitudes for the $\gamma^* p$ scattering process. The quark-proton scattering helicity amplitudes content of F_{14} was identified as [1],

$$i \frac{\bar{k}_T^1 \Delta_2 - \bar{k}_T^2 \Delta_1}{M^2} F_{14} = A_{++,++} + A_{+,-,+} - A_{-,-,+} - A_{--,-}, \quad (7)$$

where $\bar{k}_T = (k_T + k'_T)/2$, and we defined, $A_{\Lambda'\lambda',\Lambda\lambda}$, $\Lambda(\Lambda')$ and $\lambda(\lambda')$ being the proton and quark initial (final) helicities, respectively [24]. Following Jacob and Wick [25] one has,

$$A_{-\Lambda'-\lambda',-\Lambda-\lambda} = \eta_P (-1)^{\Lambda'-\lambda'-\Lambda+\lambda} A_{\Lambda'\lambda',\Lambda\lambda}^*, \quad (8)$$

η_P being the phase factor accounting for intrinsic parity and spin.

This shows explicitly that for the F_{14} contribution to the nucleon matrix elements to be non-zero, at least one pair of the helicity amplitudes must be imaginary, at variance with the other spin conserving structure functions. This circumstance was called “parity-odd” in Ref.[1], borrowing the language from the description of standard two body scattering high energy processes where in the CoM the helicity amplitudes would: 1) be real, and 2) describe a situation where the particles momenta are collinear, thus leading to a zero contribution to Eq.(7), unless a violation of parity conservation occurred as the only possible source of a non zero *real* contribution.

However, for GTMDs, by allowing for relative phases among the amplitudes, the combination that forms F_{14} can indeed be imaginary, as one simultaneously moves away from a collinear description, *i.e.* as \bar{k} and Δ are let to vary independently from one another. The specific combination of amplitudes giving rise to F_{14} is therefore consistent with parity conservation so long as one gives up the idea of the quark proton scattering occurring in a single hadronic plane. We therefore here acknowledge that it is preferable to use an alternative choice of words to “parity odd” to describe this rather complicated situation.

We further reiterate that this should not be confused with *whether the matrix element of the Lorentz structure multiplying F_{14} is parity-even or odd*; parity odd matrix elements obviously occur for many other structure functions which do not display similar pathologies in the CoM, *e.g.* the structure function g_1 measures helicity which is a pseudoscalar.

In summary, we have re-explained in more detail the question addressed in Section 2 of Ref.[1]: a combination of purely imaginary helicity amplitudes is clearly zero in the hadronic (or CoM) plane. In Ref.[1] we raised the issue

that the possibility that F_{14} is measured, or that it could represent, therefore, a viable observable for OAM, was hampered by the parity transformation properties of its nucleon matrix element. F_{14} is not observable in the general framework defined in Refs.[22, 23] because of parity constraints which limit the amplitudes to be purely imaginary, a configuration which is not physically attainable in a single hadronic scattering plane. This problem can be overcome by going beyond the framework summarized by the master formula of [22, 23], an issue that will be addressed in an upcoming publication.

Differently from helicity, which is promptly observable, the matrix element corresponding to F_{14} is parity even. Although this is the source of the measurability issue for F_{14} , it does not interfere with its identification with OAM which is also a parity even quantity. In this respect, our work was misrepresented in Ref.[5].

To clear up any confusion, the same result is obtained by using parity and Light Front (LF) parity. Under LF parity a transverse vector, $\mathbf{v} \equiv (v_1, v_2)$ undergoes $L(R) \rightarrow R(L)$ (or $v_1 \mp iv_2 \rightarrow v_1 \pm iv_2$), which takes

$$i\bar{\mathbf{k}}_T \times \boldsymbol{\Delta}_T \rightarrow -i\bar{\mathbf{k}}_T \times \boldsymbol{\Delta}_T. \quad (9)$$

Under LF parity transformations the x -components change sign but the y -components do not. Both LF parity and LF helicity amplitudes were implemented *e.g.* in chiral quark models while accounting for the zero modes [26], and more recently in Ref.[19] where for the Melosh transform, the 3-quark proton state is written with all the helicities that reverse in order to have the proton helicity reverse. In a LF parity transformation, because of the different behavior of LF helicity and 3-momenta, one has,

$$\begin{aligned} & \bar{u}(\tilde{p}', -\Lambda'_{LF}) \gamma^0 i \sigma^{ij} \gamma^0 \frac{\bar{k}_i \Delta_j}{M^2} u(\tilde{p}, -\Lambda_{LF}) F_{14}(x, 0, \bar{\mathbf{k}}_T^2, \bar{\mathbf{k}}_T \cdot \boldsymbol{\Delta}_T, \boldsymbol{\Delta}_T^2; \eta) \\ &= (\eta_P)^2 2iM \frac{(\bar{\mathbf{k}}_T \times \boldsymbol{\Delta}_T)_z}{M^2} (\Lambda_{LF}) \delta_{\Lambda_{LF} \Lambda'_{LF}} F_{14}(x, 0, \bar{\mathbf{k}}_T^2, \bar{\mathbf{k}}_T \cdot \boldsymbol{\Delta}_T, \boldsymbol{\Delta}_T^2; \eta), \end{aligned} \quad (10)$$

for which there is a plus sign times the LF helicity, consistently with what we find.

Therefore, the argument based on which it is important to use LF helicity amplitudes and parity [5] is inconsequential for the angular momentum decomposition. In fact, the matrix element defining L_{can} , since it transforms opposite to helicity, is parity even in either case. Again, the additional distinction we make here is that, notwithstanding the LF parity argument given in [5], in order for any observable to be measured with an experimental setup involving a single hadronic plane, this has to be a parity preserving object. So we are not dwelling on the parity argument but concentrating on what is/can be observable.

The recent argumentations in Ref.[5], however, prompted us to investigate the existence of observables that could be sensitive to F_{14} . A natural set of reactions that helps us settle our argument of the parity transformation property of F_{14} is to consider neutrino nucleon scattering processes which notoriously admit parity violating terms. The study of observables in manifestly parity violating processes turns out, in fact, to be crucial for settling the controversy. A complete list of such observables is given in Ref.[27] on the deep inelastic ElectroWeak (EW) structure of the nucleon along with the helicity composition of the various structure functions which are observable in both inclusive unpolarized and polarized DIS. Two of the (current conserved) structures appearing at twist two are parity violating namely the structure function F_3 in unpolarized scattering, and A_1 in longitudinally polarized scattering, while two are parity conserving, F_1 in unpolarized scattering, and G_1 in longitudinally polarized scattering. All four structure functions appear in the transversely polarized vector boson scattering process,

$$(W^\pm, Z^0) + \text{Nucleon} \rightarrow (W^\pm, Z^0)' + \text{Nucleon}'.$$

Each one corresponds to a specific combination of the helicity amplitudes, $T_{\Lambda(W,Z)\Lambda,\Lambda'_{(W,Z)}\Lambda'}$ where $\Lambda_{(W,Z)}$ and Λ are the W^\pm, Z and proton helicities, respectively.

We focus first on the parity conserving process involving G_1 which can be written as,

$$\begin{aligned} G_1 &\propto T_{+,+,+} - T_{+,-,+} - T_{-,-,+} + T_{-,-,-} \\ &= \sum_{\lambda, \lambda'} \left(g_{\lambda\lambda'}^{1,1} \otimes A_{+\lambda,+\lambda'} - g_{\lambda\lambda'}^{1,1} \otimes A_{-\lambda,-\lambda'} - g_{\lambda\lambda'}^{-1,-1} \otimes A_{+\lambda,+\lambda'} + g_{\lambda\lambda'}^{-1,-1} \otimes A_{-\lambda,-\lambda'} \right) \\ &= \sum_{\lambda, \lambda'} \left[g_{\lambda\lambda'}^{1,1} \otimes (A_{+\lambda,+\lambda'} - A_{-\lambda,-\lambda'}) - g_{\lambda\lambda'}^{-1,-1} \otimes (A_{+\lambda,+\lambda'} - A_{-\lambda,-\lambda'}) \right] \end{aligned} \quad (11)$$

where we considered factorization into quark-proton matrix elements, $A_{\Lambda'\lambda',\Lambda\lambda}$, and the subprocess,

$$(W^\pm, Z^0) + \text{quark} \rightarrow (W^\pm, Z^0) + \text{quark}'.$$

We see that, in the near collinear limit with zero mass quarks, the hard process is described by the amplitudes $g_{++}^{1,1}$ and $g_{--}^{-1,-1}$ (Fig. 1). With this reduction in amplitudes the sum becomes,

$$G_1 \propto g_{++}^{1,1} \otimes (A_{+,+;+,+} - A_{-,+;-,+}) - g_{--}^{-1,-1} \otimes (A_{+,-;+,-} - A_{-,-;-,-}) \quad (12)$$

The hard amplitudes in Eq. (12) are evaluated by introducing the quarks' electroweak current,

$$J^\mu = \bar{\psi}\gamma^\mu(g_V\mathbb{1} - g_A\gamma^5)\psi, \quad (13)$$

where $g_V = g_A = 1$ in case of charged currents (W^\pm scattering) and, for neutral current, g_V and g_A represent the weak vector and axial charges of the quarks.²

One finds,

$$g_{\pm\pm}^{\pm 1, \pm 1} = \frac{q^- \sqrt{2k^+ k'^+}}{\hat{s}} [(g'_V g_V + g'_A g_A) \mp (g'_V g_A + g'_A g_V)] \quad , \quad (14)$$

where the prime quantities indicate the final state where both the quark and the final boson can be different from the initial ones in off forward scattering. Notice that the parity even relation $g_{++}^{11} = g_{--}^{-1,-1}$, is obtained by setting “VA” or “AV” couplings equal to zero, and therefore it contains only “VV” or “AA” couplings, while the opposite happens for the parity odd relation, $g_{++}^{11} = -g_{--}^{-1,-1}$. One can write G_1 in terms of parity even and odd contributions as,

$$\begin{aligned} G_1 \propto & (g'_V g_V + g'_A g_A) \otimes (A_{++,++} - A_{-+,-+} + A_{--,--} - A_{+-,-+}) \\ & - (g'_V g_A + g'_A g_V) \otimes (A_{++,++} - A_{-+,-+} - A_{--,--} + A_{+-,-+}) \end{aligned} \quad (15)$$

where the parity even hard scattering term selects a combination of soft helicity amplitudes that has the structure of the DIS structure function g_1 (or equivalently \tilde{H} in the off-forward case, or G_{14} at the GTMD level), while the parity odd combination selects the combination that has the structure of G_{14} at the GTMD level, but that is zero when integrated over k_T or in the forward limit.

A similar analysis, for the function A_1 gives,

$$\begin{aligned} A_1 \propto & (g'_V g_V + g'_A g_A) \otimes (A_{++,++} - A_{-+,-+} - A_{--,--} + A_{+-,-+}) \\ & - (g'_V g_A + g'_A g_V) \otimes (A_{++,++} - A_{-+,-+} + A_{--,--} - A_{+-,-+}), \end{aligned} \quad (16)$$

where the role of the “VV” or “AA” couplings has been switched so that the F_{14} -like term appears now in the parity even contribution. Notice that because the parity violating terms come from the hard process and these select the helicity states by definition, at the correlator level things remain parity even.

In QCD, in the forward limit [27],

$$G_1 = \sum_q (g_V g'_V + g_A g'_A) g_1^q + \text{antiquarks} \quad (17)$$

$$A_1 = - \sum_q (g_V g'_A + g_A g'_V) g_1^q + \text{antiquarks} \quad (18)$$

where the sum is taken over the quarks flavors. This relation is valid for GPDs and TMDs as well, since $A_{++,++} - A_{-+,-+} - A_{--,--} + A_{+-,-+} = 0$.

If a parity violating term in the unintegrated and off-forward QCD amplitudes level survived, it would be seen as either a parity violating contribution, $\propto g_V g'_A + g_A g'_V$, to the unintegrated parity even G_1 type observable, or as a parity conserving contribution, $\propto g_V g'_V + g_A g'_A$, to the parity violating A_1 structure function.

² See, *e.g.*, Ref. [28] for notation.

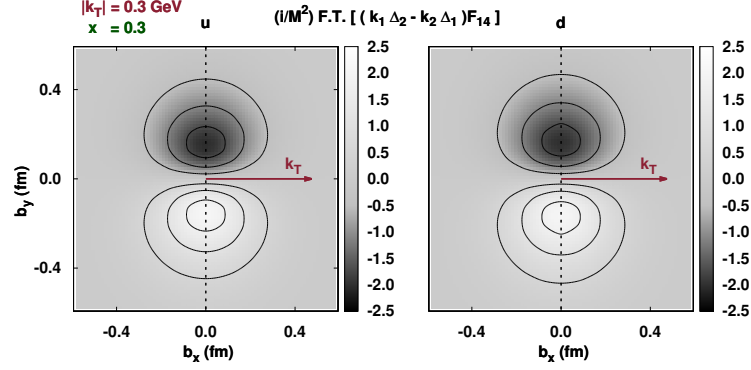


FIG. 2: Fourier transform of the correlator components defining F_{14} , Eq.(21) for $x = 0.3$, $k_1 = 0.3$ GeV, and $k_2 = 0$. The u and d quarks contributions are represented on the LHS and RHS, respectively.

Summarizing this part, to make our observation on the role of parity transformations more concretely tied to experiment, we proved that the same helicity combinations defining F_{14} and G_{11} appear among the observables for deeply virtual scattering processes as parity violating contributions to the (integrated) QCD structure functions.

To measure F_{14} and G_{11} and be consistent with the parity transformation properties in QCD one needs to define therefore an additional hadronic plane. Because F_{14} has the kinematic factor for a longitudinally polarized target going to an unpolarized quark and spectator, it is clear that the hadronization process of the active quark will involve unpolarized functions. Also, as GTMDs depend on the momentum transfer, one has to consider exclusive processes, which rules out “dihadron” fragmentation functions. What is needed is a final state particle that will force k_T to be fixed, or in some narrow range.

An exclusive process of the type: $\gamma^* + p \rightarrow \gamma + \pi^+ + \pi^- + p'$ will be required. The 4-momenta are set as $q + p = q' + p_1 + p_2 + p'$. There are 5 invariants, $s = (p + q)^2$, $t = (p' - p)^2$, $s_{12} = (p_1 + p_2)^2$, $s_{13} = (p_1 + p')^2$, $t_1 = (q - p_1)^2$. All other invariants can be written in terms of these. With this kinematical set of variables one can fix the k_T of the incoming quark, as we will elaborate on in future work.

Finally, we recall that GTMDs were originally introduced as purely theoretical tools to interpret the possible relations among TMDs and GPDs [10] before even considering them as viable means to extract physics information on OAM from experiment in their own merit [19]. In this respect, the recent developments in [1–3, 15, 19, 21] allow us to see how both canonical and kinetic OAM can be represented and compared at the density level. For the density distributions describing quarks’ canonical OAM, L_{can}^q one has,

$$\bar{u}(p', \Lambda') \frac{i\sigma^{ij} \bar{k}_T^i \Delta_T^j}{M^2} u(p, \Lambda) F_{14} \propto \langle \mathbf{S}_L \cdot \bar{\mathbf{k}}_T \times \Delta_T \rangle, \quad (19)$$

while for the kinetic one, L_q ,

$$\bar{u}(p', \Lambda') \frac{i\sigma^{ji} \Delta_T^j}{M} u(p, \Lambda) G_2 \propto \langle \mathbf{S}_L \times \Delta_T \rangle \equiv \langle \epsilon_T^{ij} \Delta_{Tj} \rangle \quad (20)$$

Both distributions describe quarks that are displaced from the origin in the transverse plane. For F_{14} , the displacement is obtained through Fourier transformation of the quark-quark correlator components defining this structure function (Ref. [19] and Eq.(6)),

$$\text{F.T.} \left(W_{++}^{\gamma^+} - W_{--}^{\gamma^+} \equiv i \frac{\bar{k}_1 \Delta_2 - \bar{k}_2 \Delta_1}{M^2} F_{14}(x, 0, \bar{\mathbf{k}}_T, \Delta_T) \right) = -\frac{1}{M^2} \epsilon_T^{ij} \bar{k}_T^i \frac{\partial}{\partial b_j} \mathcal{F}_{14}(x, 0, \bar{\mathbf{k}}_T, \mathbf{b}), \quad (21)$$

where, $\epsilon_T^{ij} = \epsilon^{-+ij}$, and,

$$\mathcal{F}_{14}(x, 0, \bar{\mathbf{k}}_T, \mathbf{b}) = \int \frac{d^2 \Delta_T}{(2\pi)^2} e^{-i\mathbf{b} \cdot \Delta_T} F_{14}(x, 0, \bar{\mathbf{k}}_T, \Delta_T), \quad (22)$$

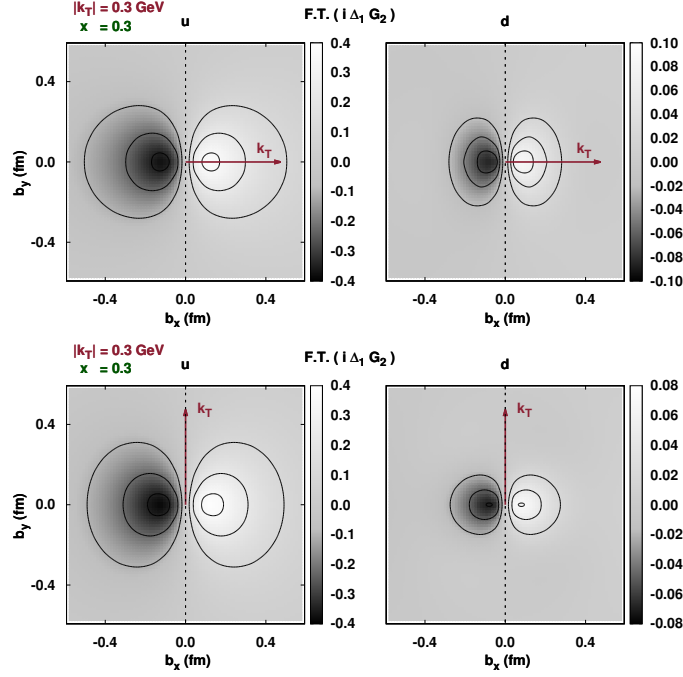


FIG. 3: Fourier transform of the correlator components defining \tilde{E}_{2T} , Eq.(23) at $x = 0.3$, for the components: $k_1 = 0.3$ GeV, and $k_2 = 0$ (upper panels); $k_1 = 0$, and $k_2 = 0.3$ GeV (lower panels). For both the upper and lower panels, the u and d quarks contributions are represented on the LHS and RHS, respectively.

Notice that $\bar{\mathbf{k}}_T$ needs to be kept at a fixed value in order to see this displacement, *i.e.* an integration over $\bar{\mathbf{k}}_T$ gives a zero result. This configuration corresponds to OAM generated through circular motion in the $x - y$ plane [19].

For the configuration corresponding to the unintegrated G_2/\tilde{E}_{2T} (for more details on the specific GTMDs see Refs.[1, 10]), the distortion in the transverse plane is described by the Fourier transform of the distribution,

$$\text{F.T.} \left(W_{++}^{\gamma^i} - W_{--}^{\gamma^i} \equiv -i\epsilon_T^{ij} \frac{\Delta_j}{M} \tilde{E}_{2T}(x, 0, \bar{\mathbf{k}}_T, \Delta_T) \right) = -\frac{1}{M} \epsilon_T^{ij} \frac{\partial}{\partial b_j} \tilde{\mathcal{E}}_{2T}(x, 0, \bar{\mathbf{k}}_T, \mathbf{b}) \quad (i, j = 1, 2), \quad (23)$$

where we used the notation of [10]. In this case \mathbf{k}_T can be parallel to Δ_T , so Eq.(23) gives a non zero result when integrated over \mathbf{k}_T , and OAM points in the direction orthogonal to both S_L and Δ_T , consistently with the representation given in Ref.[29]. Notice that, the distribution functions in principle depend on a vector n that comes from the gauge link [10]. The choice of n is dictated by the leading contribution of the correlator in view of the factorization theorems. That is, the light-cone direction is selected by the hard probe and allows for a separation into hard and soft contributions. The probability density of the unpolarized PDF is valid, as it is well known, only in the light-cone gauge, which masks the dependence on that vector. Once that direction is fixed, the Dirac structure depends on it and the resulting distribution functions will reflect the photon's perspective. We agree that, in such cases, the z -direction cannot be changed arbitrarily; an arbitrary rotation of the z -axis was shown to lead to incomplete results in the past [30]. However, for the GTMDs there is no known probe that could disentangle the interplay of the $(x, \xi, \bar{\mathbf{k}}_T, \Delta_T, \bar{\mathbf{k}}_T \cdot \Delta_T)$ variables. Hence, the vector n is unconstrained and we believe that it can be freely chosen. For coherence in defining the GTMDs limit to TMDs or GPDs, the authors of Ref. [10] explicitly chose n to be the light-cone vector $n^\mu \equiv (1; 0, 0, -1)/\Lambda$ to properly reproduce the path of the appropriate Wilson link.

In Figure 2 and Figure 3 we show the distributions in the transverse plane corresponding to Eq.(21) and to the $i = 2$ component of Eq.(23), respectively. All functions were obtained in the reggeized quark-diquark picture of Ref.[31, 32]. In Figure 4 we show the integrated over \mathbf{k}_T distribution, for the same value, $x = 0.3$ as in Fig.3. A similar plot could not be drawn for F_{14} , since this integrates to zero, thus indicating that these two quantities, although they both represent OAM, correspond to profoundly different partonic configurations.

In conclusion, we have analyzed the issue of observability of both canonical and kinetic OAM. Canonical OAM

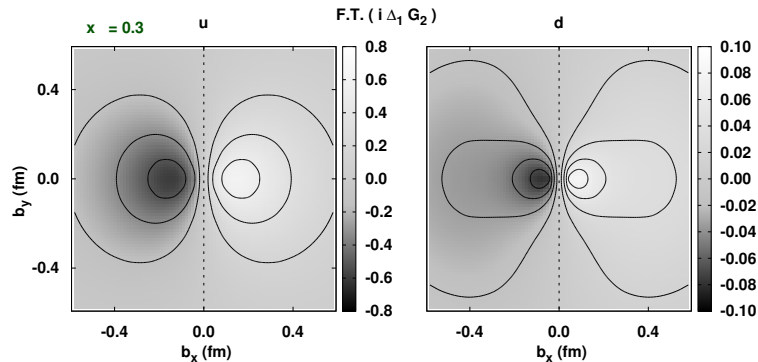


FIG. 4: Fourier transform of the u -quark (L) and d -quark (R) correlator components defining \tilde{E}_{2T} , integrated over \mathbf{k}_T , at $x = 0.3$.

can be identified with the second moment in k_T of the GTMD F_{14} , a formal proof that such quantity is related to OAM having been given in Ref.[15]. The observability of OAM is however hampered by the fact that F_{14} , and an analogously a GTMD in the axial-vector sector, G_{11} , cannot be connected to any of the GPDs and/or TMDs, thus making it challenging to define physical processes which would be sensitive to these quantities. In spite of the fact that non-zero results for F_{14} can be obtained from either models, or by direct calculations on the lattice, a process that selects this quantity has not been yet identified. The physical content of the models, whether these are “perturbative” or arise from “effective field theories” cannot be taken as a proof of the existence of an observable. However, we notice that each of the models explored so far giving a non-zero result for F_{14} , carry some remnant of confinement, while only one model calculation that clearly does not have confinement - the quark-target model - (as an “ensemble of free quarks”, with no gluon across the vertices) gives zero for F_{14} (see analogous calculation in Ref.[33] on g_2). We take this as an indication that the gauge link structure of F_{14} plays a fundamental role, as already suggested in [21] and that, looking at future studies, its connection with the final state interactions implicitly present in the twist three definition of OAM through G_2/\tilde{E}_{2T} will give key information on the nature of OAM.

Finally, we reiterate that in our analysis, while reinforcing the use of the LF, we give a physical motivation for the fact that F_{14} has not yet been associated to any observable, that goes beyond simply stating the issue [10, 19]. Our explanation is founded on the transformation properties of the unintegrated correlator under parity which do not allow for the specific combination of helicity amplitudes generating F_{14} to be observed in any given single hadronic plane. This prompts the derivation of an extension of the “master formula” used so far to describe both semi-inclusive and exclusive lepton-proton scattering [22]. At the same time we point out that the transformation property of the matrix element associated with F_{14} is a distinct issue that should not be confused with the observability of the quantity through its decomposition in quark-proton helicity amplitudes. In this respect, F_{14} is consistent with the transformation under parity of OAM, and we do not contend the opposite.

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